**Fluid and Liquid Dynamics**

**Chapter 1 – Introduction**

**States of Matter** – There are three states of matter:

**Solids** – These are substances which have a definite shape.

**Liquids** – These are substances which we can see but cannot hold and deform instantly

**Gases** – These are substances that have no definite shape and which cannot be normally seen.

Substances exist at all times in at least one of these states. It is possible to have a substance coexist in more than one phase.

Substances are made up atoms, and a molecule is the smallest possible quantity that still retains its properties. For elements the smallest possible quantity is an atom that still has the properties.

Engineers have to be well versed in the properties of substances because they have to be able to apply their knowledge in order to construct and design objects.

**Chapter 2 – Basic Concepts and Principles**

There are **two** **different** **scales** **of** **measurement** that are used. **Microscopic** and **Macroscopic** scale. Microscopic are measurements that beyond the capacity for the human eye to see, and macroscopic are measurements that deal with objects large enough for perception of the human eye.

There are three **fundamental** **Dimensions**. These dimensions are used to **derive** **and** **calculate** **other** **parameters** or properties. The three dimensions are:

**Length [L]** - This is any form of measureable distance.

**Mass [M]** - This is the amount of matter involved.

**Time [T]** - This is the amount of time involved in the calculation.

From these three dimensions, variables and parameters such as velocity, Force, Acceleration, Density, and Viscosity can be derived. There are some other dimensions that are used for other properties these include:

**Current** - Used in equations using electricity

**Temperature** – Used in thermodynamics

**Amount [mol]** – The amount of molecules of the substance

**Light Intensity** – Amount of lumens.

In all equations, in order to be valid, it is said that the equation must be **dimensionally** **homogeneous**. By this it means that all **terms involved must be made up of the same fundamental Dimensions.**

There are two major standards of measurement. These standards are:

**SI** – Standard System of Units (**Metric** **System**)

**Imperial** **System** – Inches, Lbs, Fahrenheit, Miles

There are some **laws** **of** **conservation** that are known. These are:

**Law of Conservation of Mass** – Mass cannot be created or destroyed.

* An isolated system’s **mass** **remains** **constant**.
* Accumulation of mass accounts for mass that doesn’t get output.
  + - **Mass Accumulation = Mass Input – Mass Output**
* For systems where Chemical reactions occur Accumulation is:
  + **Mass Accumulation = Mass Input – Mass Output + Mass Generated**
  + **Mass Accumulation = Mass Input – Mass Output – Mass Consumed**
    - Note that Mass can be generated or consumed through Reactions

**Law** **of** **Conservation** **of** **Energy** – Energy cannot be created or destroyed only Transformed.

* An isolated system’s **energy** **remains** **constant**.
* Energy can exist in **different** **forms** in an isolated system (kinetic, chemical, etc…)
* Accumulation of energy can occur when a energy is flowing through a system
  + **Energy Accumulation = Energy Input – Energy Output**

**Law of Conservation of Mass-Energy** – In nuclear reactions **mass** **can** **be** **converted** **to** **energy**.

* In a nuclear reaction the **Mass**-**Energy** **of** **the** **system** **remains** **constant**.
* Accumulation of Mass-Energy can be calculated in a system by:
  + **Accumulation of Mass-Energy = Mass-Energy Input – Mass-Energy Output**

**Law of Conservation of Momentum** – Momentum cannot be gained or lost

* In an isolated system, in the absence of force **momentum** **remains** **constant**.
* In a **completely** **Elastic** **Collision** **Momentum** **remains** **constant**.
* Individual objects in the system can have different momentums but the net momentum does not change.

There are **different** **measurements** **of** **Temperature**. The important scales are:

**Kelvin** – This scale is a natural scale in that it does not have any negative values.

* It starts at 0 at which temperature molecules do not move. This is referred to as absolute 0.

**Celsius** – This scale is based around the reference points of water. It has negative values in it, and it does is based around the melting and boiling points of water. 0 is the freezing point of water, and 100 is the boiling point of water. In relation to Kelvin it is (273.15 + C) 1K/1C.

**Fahrenheit** – This scale is also based around the reference points of water, except that water’s melting point is 32 degrees, and the boiling point 212. It is related to Celsius by (9/5 C + 32).

As with all units, if they are not the same then they cannot be added or subtracted. This means that equations which use certain units must be calculated using those proper units. All conversions must be done prior to adding them.

**Chapter 4 – Equilibrium between Phases**

When talking about substances we have different ways of describing them. We can describe their **composition**, their **phases**, and their **rate** **of** **change** **or** **equilibrium**.

When describing their **composition** is mainly referring to the **percentage** **or** **fraction** in which one or more substances in a mixture takes up.

**Composition** – This is the **number** **of** **parts** **in** **which** **one** **type** **of** **substance** **makes** **up** **a** **Mixture**. There multiple ways in which to describe composition: Mass Fraction, Mole Fraction, and Volume Fraction.

**Mass** **Fraction** – This is the percentage of mass which a substance makes up in a mixture.

**Mole** **Fraction** – This is the percentage of Molecules which a substance makes up in a mixture.

**Volume** **Fraction** – This is the percentage of space that a substance takes up in a mixture.

For all types of composition it calculated the same way:

**Wf = wp / wt**

Where Wf is the Fraction of the composition, wp is the value of the substance, and wt is the total value of the mixture.

This composition is a % or ratio, which means that it is not related to the size of the system. Whether the system is 10kg or 100kg the composition will be the same so long as the system is proportional. Another thing to note is that the **Fractions** **have** **to** **add** **up** **to** **1 or 100%** because it represents the whole system.

It is quite easy to convert from Mole Fractions to Mass fractions. You just have to multiply the percentage by the total amount, and then convert that using molar mass. This will give you the amount in the new unit, and then you can divide it by the total amount of those units to get it to the new %.

Another way of describing the substance is stating which phase it is currently in.

**Phase** – A **completely** **homogeneous** **substance** **and** **uniform** **state** **of** **matter**. This means that all the molecules of the substance are attracted to each other in certain fashion based on certain factors.

Another statement that can be made about substances is that they are at equilibrium.

**Equilibrium** – **A system is said to be at equilibrium if it is at a** **perfect** **balance** **between** **all** **changes**. The change forward is going at the same rate as the change backward. This means that as a whole, the system undergoes no change.

There is a relationship that can be stated about phases and equilibrium. This is called the phase rule.

**Phase** **Rule** – **F + P = C + 2**

**Degrees of Freedom (F)** – The **# of parameters (Intensive Variables)** you can set. (Pressure, Temperature, etc)

* + **Intensive** **variables** are variables which do not depend on the size of the phases. So they are relative to the amount available. Thus Volume and Mass aren’t, but the ratios are because the ratio doesn’t depend on the size.(ex. Density)
  + Some common intensive variables are:
    - **Temperature**
    - **Pressure**
    - **Specific Volume / Density**
    - **Mass or Mole Fraction**

**Phases (P)** – This is the **number** **of** **phases** present in the system. (liquid, solid, gas)

**Components (C)** – This is the **number** **of** **components** in the system (Salt, Coffee, etc)

The phase rule is used in order to determine the number of phases that can coexist at equilibrium.

**SINGLE COMPONENT SYSTEMS:**

When there is only one component it can be said that three possibilities exist based on the phase rule.

* **1 Phase**
  + There are two degrees of freedom which means that two variables can be set without changing the number of phases present in the system.
* **2 Phases**
  + There is one degree of freedom which means that one intensive variable can be set without changing the number of phases.
* **3 Phases** 
  + There are no degrees of freedom with three phases and that means that no variables can be set without changing the numbers of phases. This means that each component has **at** **least** **one** **point** **in** **which** **all** **three** **phases** **coexist**. This is known as the **Triple** **Point**.

**SINGLE COMPONENT PHASE DIAGRAMS:**

When looking at systems of components it is best to represent their equilibrium with a **phase** **diagram**. The two most common types for single component systems are **P-V (Pressure vs. Specific Volume)** and **P-T (Pressure vs. Temperature).**

**P-T Diagram**

* These graphs show the locations of the different phases of the component or Mixture.
  + For a single phase you have **Solid**, **Liquid**, **Vapour** regions only. The points in which more than one phase exists are seen as lines called **two phase lines** separating the phase regions, or the **triple** **point** when dealing with three phases.
* Another important point that exists on the graph is the **Critical** **Point**.
  + This point is the point in which **liquid** **and** **vapour** **cannot** **be** **distinguished**. When the **temperature** **and** **pressure** are past this point it is said to a **gas**.

When looking on a P-V or P-T graph you can have Horizontal straight lines called **Isobars** in which the **pressure is kept constant**.

* This means that when the pressure is kept constant but the Specific Volume or Temperature is changed then the position on the Isobar will change and a change in phases can occur if the position based on the Temperature or Specific Volume goes into a new Phase Region.

Similar to Isobars you can have **Isotherms** which are when the pressure or Specific Volume is changed but the **Temperature remains constant.**

* When the temperature is kept constant but the pressure is changed you can undergo a phase change. This happens because as you increase the pressure you move along the Isotherm and this can move into a new phase region.

**P-V Diagram**

While on P-T Diagrams you have a very basic set up, P-V diagrams are a little more complex. You can have **phase regions which have more than one phase**.

* When you have a **phase** **region** **with** **two** **phases** you have an **equilibrium** **at** **which** **the** **two** **phases** **coexist**. These are called **two** **phase regions**. (liquid + vapour) or (liquid + solid) or (vapour + solid)
* There is a line called the **triple phase line**. This line is the point where **three phases coexist**. The **Pressure and Temperature Remain Constant** while the specific volume changes. On the P-T graph this is the **Triple** **Point**
* There are a few more lines of importance **Bubble** **Point**, **Dew** **Point**, **Saturation**.
  + - The **Saturation** **Curve** is the line at which the two phase region (V + L) becomes Vapour and vice-versa.
    - The **Bubble** **Point** **Curve** is the line on the saturation curve at which the **first** **molecule** **of** **gas** **appears** in the system. This is the beginning of the two phase region.
    - The **Dew** **Point** **Curve** is the line on the saturation curve at which the **first** **molecule** **of** **liquid** **appears** in the system. This is the beginning of the two phase region.
* The **Critical** **Point** is the **maximum of the saturation curve**. It is this point where the bubble point curve meets the dew point curve. The **density** **(and specific volume) of the** **liquid** **is** **practically** **the** **same** **as** **the** **density** **of** **the** **gas** and is said to be equal.
  + - The temperature which goes through the critical point is called the **critical** **temperature** and the pressure is called the **critical** **pressure**. Anything above this point is considered to be a gas.

**TWO COMPONENT SYSTEMS:**

Systems that are made up of two components are called **Binary Mixtures.** Based on the phase rule it can be said that binary mixtures have three possibilities:

**1 Phase – Degrees of Freedom is 3**

**2 Phases – Degrees of Freedom is 2**

**3 Phases – Degrees of Freedom is 1**

Because there are too many degrees of freedom for us to study one is usually fixed then it is possible to study the relation between the other two variables.

In order to break it down even more we focus on the relationship between Vapour and Liquid phases, and liquid and solid systems.

**Vapour + Liquid Systems:**

On a **T-X** **diagram** the left side represents 100% of one component and the right side represents 100% composition of the other component. There are three different possible systems between Vapour and Liquid in a Binary Mixture.

* **Completely** **Miscible** **Liquids**
  + - There are three Phase Regions ( Liquid, V+L, Vapour)
    - You have the Bubble Point Curve on the Bottom
    - You have the Dew Point Curve on the Top
    - If you draw a tie line across the system the Temperature remains constant while the mass/mole fraction changes. This can lead to a change of phases.
* **Completely** **Immiscible** **Liquids**
  + - You have four phase regions. (L1 + l2, L1 + V, L2 + V, V)
    - The liquids do not mix and as such they always separate from each other.
    - You can draw a tie line across the system and this will adjust the system’s phases.
* **Partially** **Miscible** **Liquids**
  + - You have six Phase Regions (L1, L1 + l2, L2, L1 + V, L2 + V, V)
    - The liquids are soluble at certain percents, but not at others.
    - Ties lines can be drawn across these systems.

**Liquid + Solid Systems:**

You can have three types of systems.

* **Completely** **Miscible** **Solids**
* **Partially** **Miscible** **Solids**
* **Completely** **Immiscible** **Solids**

The set up is the exactly same as with liquid and vapour systems, however instead of dew point and bubble point curves you have **Solidus** and **Liquidus** **lines**.

* **Solidus** **Lines** are lines in which the first molecules of solids appear in the system
* **Liquidus** **lines** are lines in which the first molecules of liquids appear in the system

**LEVER RULE:**

For all **two** **phase** **regions** (both 1 and 2 component systems) you need to be able to figure out the **composition** **of** **the** **system** at a given point. When in a single phase region you can just read the composition directly off the graph, but when you are dealing with two phases you have to use a technique called the **lever** **rule**.

* The lever rule is used to calculate the composition of a phase (or component of a phase) in a two phase region.
* The rule is:
  + - **Wm = ( wx – w2 ) / ( w1 – w2 )**
    - Wm is the composition of the component/phase
    - wx is the point on the tie line or Isobar.
    - w1 is the phase’s Composition which you are not calculating.
    - w2 is the phase’s Composition which you are calculating.

This is very helpful when having to calculate values at a given point (Mass, Volume, etc) which you don’t know directly.

There can sometimes be a point on a graph called an **Azeotrope**.

* This point is where the bubble point (or Liquidus Line) and dew point (or Solidus Line) on a binary mixture touch. At a specific temperature the **composition** **of** **two** **phases** **is** **practically** **the** **same**. This is called the **Azeotropic** **Composition**.

Another point that can occur in Liquid-Solid binary systems is the **Eutectic** **Point**.

* This point is the where the **freezing** **point** **is** **the** **lowest**. This is where the composition and Temperature are specific.
* Compositions to the **right** **of** **the** **eutectic** **point** are called **hypereutectic**
* Compositions to the **left** **of** **the** **eutectic** **point** are called **hypoeutectic**

**Chapter 5 – Ideal Gas Mixtures**

Dalton stated that the pressure of the system was equal to the combined pressures of the individual gases in the system.

**Dalton’s Law – P = Pa + Pb Pi = YiP**

* P is the total pressure of the system
* Pa is the pressure of gas 1
* Pb is the pressure of gas 2
* Pi is the partial pressure of a gas in the system
* Yi is the mole fraction of the gas

Amagat stated that the volume of the system was equal to the combined volumes of the individual gases in the system.

**Amagat’s Law – V = Va + Vb**

* V is the total volume occupied by the system
* Va is the volume occupied by gas 1
* Vb is the volume occupied by gas 2

When dealing with Ideal Gases a few Axioms must be assumed.

* An Ideal Gas has mass, but its individual molecules do not occupy any volume.
* A molecule does exert any attractive or repulsive forces on neighbouring molecules.

In order to understand the behaviour of gases researcher came up with **equations** **of** **state**.

* An **equation** **of** **state** (**EOS**) is a relationship between multiple intensive variables.

**EOS’s for Ideal Gases:**

**Boyles Law – PV = c P1V1 = P2V2**

* P is the pressure of the gas
* V is the volume of the gas
* c is the constant for that gas at a constant temperature

**Charles Law – V/T= c V1 / T1 = V2 / T2**

* V is the volume of the gas
* T is the Temperature of the gas
* C is the constant at a constant Pressure

**Ideal Gas Law – PV = nRT PV­m = RT**

* P is the pressure of the gas
* V is the Volume of the gas
* n is the number of moles of the gas
* T is the Temperature of the gas
* R is the universal Gas Constant (8.314 (kPa m3)/(kmol K)
* Vm is the Molar Volume (V/n)

**Kinetic Theory of Gases:**

According to the Kinetic Theory of Gases:

* The volume of the molecules is small compared to the volume occupied by the gas and is negligible.
* Molecules have no forces between them.
* Molecules move at random, and all collisions between molecules are perfectly elastic, thus no momentum or kinetic energy is lost.

**Pressure according to the kinetic Theory:**

* Since the collisions are assumed to be elastic then the change in momentum of a collisions will be equal to . And since the number of collisions over a period time is equal to . This means that changes in momentum over time is equal to
* Since this is the change in momentum of one direction, the change of the momentum of all directions would be which is the same as the resultant vector’s change. Thus the change in momentum of a molecule over time is (). Where is the vector’s velocity.
* Force is equal to the change in momentum thus: and since pressure is the Force over Area: . This is the pressure that one molecule exerts, so in order to get the full pressure of the system you would multiply it by the number of molecules, thus .
* However, is the same as volume, thus

Using the kinetic Theory of Gases it can be shown that the law previously mentioned (Boyles, Charles, etc…) are true.

The **Average** **velocity of a molecule** is called the **Root** **Mean** **Square** **Velocity** and is equated to:

* is the root Mean Square Velocity based on Vector **c2**
* R is the universal gas Constant
* T is the Temperature
* M is the molar mass

The **Most** **Probable** **Velocity** is the velocity of a molecule that is **most** **likely**.

The **Mean** **Velocity** is the velocity of a molecule which is the **average** **velocity of all molecules**

Another useful parameter that can be found when dealing with ideal gases is **Mean Free Path**.

* Mean Free Path is the **average** **distance** **between** **molecular** **collisions**.
* The mean Free Path is calculated by dividing the distance travelled by the # of collisions.
* The # of collisions is calculated by multiplying the number of molecules per volume, by the volume of the container.
* The Mean free path is:

We can also find the relationship between temperature and Pressure and the Mean Free Path.

* The Mean Free Path in relation to Temperature and Pressure:

**ANGSTROM IS EQUIVALENT TO 10-10 M**

**Chapter 6 – Real Gases:**

**Van der Waals Equation of State:**

Back in the 1800’s Van der Waals came up with an equation that he hoped would better predict the interactions of real gases. In his equations he had two constants and which were included to make up for certain discrepancies which he felt were due to a pressure and volume loss due to the intermolecular forces and volume taken up by the molecules respectively.

Or

Due to the fact that there are two constants and you have to solve for them. They can be solved from **Critical Pressure** and **Critical Temperature**:

or

and

When you are in the cubic form and are forced to solve for you have to solve for the roots. These roots represent different things are different temperatures:

When there will be **3 real roots**, and the **largest** root represents the **Dew Point Value (Vapour Pressure)** , and the **Lowest** will represent the **Bubble Point Value(Liquid Pressure).**

When there will be **3 roots**, but **two will be imaginary**, and as such there is only **one** **real** **root**. This root is the one that we want, and it represents the of the gas.

When there will be **three identical roots**.

**Corresponding Equation of States:**

Other equations that were used to in explaining the interactions of real gases was the inclusion of a **compressibility** **factor** (**Z**) which is a proportionality constant which is different for each gas. It is them placed in the ideal gas equation.

The compressibility factor can be found through two methods: either by using a **general compressibility** **chart** or through the use of **Pitzer-Curl Tables**.

**General Compressibility Charts:**

The general compressibility charts require that you know **reduced** **Temperature** () and **reduced** **pressure** () which can be calculated using the following equations:

It is said the reduced values are the ratio of the current conditions in relationship to the critical values.

**Pitzer-Curl Method:**

For the Pitzer-Curl method **accentricity** () is also required:

can be calculated using Pitzer-Curl **tables** and they are based on the reduced Temperature and Reduced Pressure.

**Mixtures:**

For mixtures the above equations can be used, however, since there is more than one substance involved a **pseudo-critical** set of values must be found. These pseudo-critical values represent a **fictitious fluid** that would **theoretically have the same properties as the mixture.**

They can then be used in place of and in the equations. However, when using the Van der Waals EOS you have to find the constants and for the mixture.